Anomalous rotational-alignment in N=Z nuclei and residual neutron-proton interaction

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Recent experiments have demonstrated that the rotational-alignment for the $N = Z$ nuclei in the mass-80 region is considerably delayed as compared to the neighboring $N \neq Z$ nuclei. We investigate whether this observation can be understood by a known component of nuclear residual interactions. It is shown that the quadrupole-pairing interaction, which explains many of the delays known in rare-earth nuclei, does not produce the substantial delay observed for these $N = Z$ nuclei. However, the residual neutron-proton interaction which is conjectured to be relevant for $N = Z$ nuclei is shown to be quite important in explaining the new experimental data.

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The major advancements in the γ-ray detecting systems has made it possible for the first time to measure the high-spin states of the $N = Z$ nuclei, ^{72}Kr, ^{76}Sr and ^{80}Zr [1,2], to a spin region where the rotational-alignment of nucleons is normally expected. A comparison of the new data with the neighboring $N \neq Z$ nuclei shows that the alignment is consistently and considerably delayed for these $N = Z$ nuclei. This delay cannot be explained [3] using the cranked shell model approach which has been a powerful tool for studying the high-spin phenomena.

The proton-rich mass-80 nuclei exhibit many phenomena that are quite unique to this mass region. Unlike the mid-rare earths and actinides that have a very stable deformation, the structure changes are quite pronounced among the neighboring nuclei in the mass-80 region. Indeed, recent experiment [4] found much reduced transition quadrupole moments above the alignment process in ^{74}Kr, indicating a dramatic shape change along the yrast sequence.

For the $N \approx Z$ nuclei, there has been an open question whether the neutron-proton (n-p) correlations play a role in the structure analysis. The existence of n-p pairing in nuclear systems, in particular in nuclei with equal number of neutrons and protons, is one of the topical issues. It has been demonstrated using the cranking approaches in a single-j shell [5–8] that the rotational-alignment properties will be modified by the residual n-p interaction. In particular, it has been shown that the rotational-alignment of nucleons will be delayed for $N = Z$ nuclei as compared to the neighboring $N \neq Z$ nuclei. However, the models used in these studies are very schematic and the results of these calculations cannot be compared with the experimental data. It is also not very apparent from these calculations which component of the n-p residual interaction is responsible for the delay in the rotational-alignment.

Recently, we have performed a systematic investigation for the yrast properties of the proton-rich Kr, Sr and Zr nuclei [9]. The analysis was carried out using the projected shell model (PSM) approach [10]. We calculated the $N = Z, Z + 2$ and $Z + 4$ isotopes and studied their moments of inertia, transition quadrupole moments and g-factors. Quantitative comparisons were made with available data, and predictions were given for quantities that were not measured. The variations along the isotopic and isotonic chains were attributed to the mixing of various configurations of the projected deformed Nilsson states that are a function of shell filling.

A comparison of our PSM calculations in ref. [9] for the three Kr-isotopes ^{72,74,76}Kr is shown in Fig. 1, now including the new data of ^{72}Kr [1]. It is evident from the figure that, although, a reasonable agreement can be seen for ^{74}Kr [4] and ^{76}Kr [11] isotopes, a clear disagreement in the plot of moment of inertia occurs for the $N = Z$ nucleus ^{72}Kr. The calculated moment of inertia for the yrast states of ^{72}Kr shows a sharp backbend which is in contradiction with the smooth curve obtained experimentally. The experimental moment of inertia indicates a slight upbend around $\hbar\omega = 0.8$ MeV. The rotational-alignment is therefore significantly delayed as compared to the neighboring ^{74,76}Kr isotopes. It is known that the rotational-alignment may be delayed in cases with larger deformation, but this argument cannot be applied here since both the theoretical calculations and the experimental low-spin transition energies suggest that the deformation in ^{72}Kr does not differ very much from those of the neighboring ^{74,76}Kr.

The purpose of the present paper is to explore how the behavior of moment of inertia for ^{72}Kr can be understood in the PSM framework. In particular, we would like to see whether the observation can be explained by a known type of nuclear residual interactions. As mentioned earlier, the single-j shell models indicate that the n-p residual interaction may be responsible for the delayed rotational-alignment observed in $N = Z$ nuclei. We would like to investigate the behavior of the moment of inertia as a function of the strength of the n-p interaction employed in the PSM approach.

The description of the PSM calculations for Kr-, Sr- and Zr-isotopes can be found in ref. [9]. We would just like to mention that in the PSM calculations for these nuclei, three major shells ($N = 2, 3, 4$) for both neutron and proton are used and the shell model space includes the 0-, 2- and 4-quasiparticle (qp) states:

$$|\phi\rangle_\kappa = \left\{ |0\rangle\,,\ \alpha^\dagger_{n_i}\alpha^\dagger_{n_j}|0\rangle\,,\ \alpha^\dagger_{p_i}\alpha^\dagger_{p_j}|0\rangle\,,\ \alpha^\dagger_{n_i}\alpha^\dagger_{n_j}\alpha^\dagger_{p_i}\alpha^\dagger_{p_j}|0\rangle \right\}, \tag{1}$$

where $\alpha^\dagger$ is the creation operator for a qp and the index n (p) denotes neutron (proton) Nilsson quantum numbers which run over the low-lying orbitals below the cut-off energy. The deformed single particles are generated by the Nilsson calculation [12]. It is important to note that for the $N = Z$ nuclei under consideration, unperturbed 2-qp states of $\alpha^\dagger_{n_i}\alpha^\dagger_{n_j}|0\rangle$ and $\alpha^\dagger_{p_i}\alpha^\dagger_{p_j}|0\rangle$ with the same configuration can occur pairwise as nearly degenerate states.

The Hamiltonian employed in the PSM calculation contains the separable forces and can be expressed as $\hat{H} = \hat{H}_\nu + \hat{H}_\pi + \hat{H}_{\nu\pi}$, where H_τ ($\tau = \nu, \pi$) is the like-particle pairing plus quadrupole Hamiltonian, with the inclusion of quadrupole-pairing,

$$\hat{H}_\tau \ = \ \hat{H}^0_\tau - \frac{\chi_{\tau\tau}}{2}\sum_\mu \hat{Q}^{\dagger\mu}_\tau \hat{Q}^\mu_\tau - G^\tau_{\rm M}\hat{P}^\dagger_\tau \hat{P}_\tau - G^\tau_{\rm Q}\sum_\mu \hat{P}^{\dagger\mu}_\tau \hat{P}^\mu_\tau, \tag{2}$$

and $\hat{H}_{\nu\pi}$ is the n–p quadrupole–quadrupole residual interaction

$$\hat{H}_{\nu\pi} \ = \ -\chi_{\nu\pi}\sum_\mu \hat{Q}^{\dagger\mu}_\nu \hat{Q}^\mu_\pi. \tag{3}$$

In Eq. (2), the four terms are, respectively, the spherical single-particle energy, the quadrupole-quadrupole interaction, the monopole-pairing, and the quadrupole-pairing interaction. The interaction strengths $\chi_{\tau\tau}$ ($\tau = \nu$ or π) are related self-consistently to the deformation ε_2 by

$$\chi_{\tau\tau} \ = \ \frac{\frac{2}{3}\varepsilon_2(\hbar\omega_\tau)^2}{\hbar\omega_\nu\langle\hat{Q}_0\rangle_\nu + \hbar\omega_\pi\langle\hat{Q}_0\rangle_\pi}. \tag{4}$$

Following ref. [10], the strength $\chi_{\nu\pi}$ is assumed to be

$$\chi_{\nu\pi} \ = \ (\chi_{\nu\nu}\chi_{\pi\pi})^{1/2}. \tag{5}$$

Similar parameterizations were used in many of the earlier works [13].

The interaction strengths in Eq. (2) are consistent with the values used in the previous PSM calculations for the same mass region [9,14]. The monopole pairing strength G_M is taken to be $G_M = [18.0 - 14.5(N-Z)/A]/A$ for neutrons and $G_M = 14.5/A$ for protons. The quadrupole pairing strength G_Q is assumed to be $G_Q = \gamma G_M$, the proportionality constant γ being fixed to 0.16.

The eigenvalue equation of the PSM for a given spin I takes the form [10]

$$\sum_{\kappa'}\left\{H^I_{\kappa\kappa'} - E^I N^I_{\kappa\kappa'}\right\}F^I_{\kappa'} = 0. \tag{6}$$

The expectation value of the Hamiltonian with respect to a "rotational- band κ", $H^I_{\kappa\kappa}/N^I_{\kappa\kappa}$ defines an unperturbed band energy and when plotted as functions of spin I, it is called a band diagram [10]. Valuable information can often be extracted from the unperturbed bands before the configuration mixing. For example, crossing of two unperturbed bands near the yrast line is usually of particular interest. The results after band mixing depend on how the bands cross (crossing with large or small crossing angles) and how they interact with each other (types of residual interactions) at the crossing region.

Band-crossings can be visualized in a band diagram, for example, those between the ground-band (g-band or 0-qp band) with 2-qp bands usually consisting of the (decoupled) high-j particles. Delayed rotational-alignment may be related to unusual types of interactions. The n-p pairing is thought to be relevant here because one is dealing with the $N = Z$ systems. However, the quadrupole pairing is also a type of residual pairing interaction used in nuclear structure calculations [15], and it has been known that this force can delay a rotational- alignment process [16].

In ref. [17], it was demonstrated that the quadrupole pairing force in the Hamiltonian of Eq. (2) is crucial to shift the rotational-alignment in the rare earth region. For the case ^{175}Ta, by increasing the strength constant γ from 0.16 to 0.24, a pronounced delay of rotational-alignment was obtained, which successfully explained the data. Ref. [18] further showed the systematic results for a large number of Lu, Ta, Re and Ir isotopes, with the overall agreement with data being very satisfactory. Generally, the larger the delay of crossings, the larger the quadrupole pairing force. Thus, these results [17,18] suggested that from the study of rotational-alignment, one could extract information about the quadrupole pairing interaction in the rotating systems.

In the present study, we would like to investigate whether similar calculations with adjusting the quadrupole pairing force can reproduce the observed alignment delay in $N = Z$ nuclei. In Fig. 2, we show the results for ^{72}Kr with various quadrupole pairing strengths γ. In the calculations, γ is allowed to vary from 0.16 to 0.28, and $\gamma = 0.16$ was the original value used in calculations in ref. [9]. It can be seen that we have obtained a delay in crossing frequency, but the amount of delay is too small to explain the data. It should also be emphasized here that a sharp backbend in the plot remains for all different values of γ, in a total disagreement with the new data. Thus, we conclude that the quadrupole pairing effect is not the primary source for the delayed alignment in ^{72}Kr.

Next, we would like to study the effect of the neutron-proton interaction. As already mentioned, several theoretical calculations suggest that the residual n-p interaction may be important in the discussion of the high-spin phenomena in $N = Z$ nuclei. Specifically for $N = Z$ nuclei, the pairwise occurrence of neutron and proton 2-qp bands in the band-crossing region suggests the neutron-proton type interaction can be very important for these nuclei. Together with the g-band, it constitutes a band-crossing picture involving three bands. This is in contrast with the common picture of two-band crossing for most cases in $N \neq Z$ nuclei. The sharp backbend obtained in the PSM approach using the standard parameters may suggest that the neutron-proton interaction used is too weak. Here, we would like to investigate in a purely phenomenological manner the influence of the n-p interaction, through the n-p QQ term (see Eq. (3)) in the PSM, on the behavior of the moment of inertia. It should be noted that the strengths of the proton-proton and neutron-neutron QQ in Eq. (2) are fixed through the self-consistency condition, Eq.(4), and therefore cannot be changed. The strength of the n-p QQ given by Eq. (5) is based on the assumption of the iso-scalar coupling. This assumption may not be valid in general, and may be modified by the residual n-p interaction.

The calculations for three $N = Z$ nuclei ^{72}Kr, ^{76}Sr and ^{80}Zr are presented in Fig. 3. For each of these nuclei, we increase the strength $\chi_{\nu\pi}$ by multiplying a factor $\alpha = 1.1$, 1.2 and 1.3. A rather pronounced effect can be seen for ^{72}Kr: the increase of the n-p interaction has a combined effect of delaying crossing frequency and smoothing the backbending curve. Eventually, the basic feature in the data can be described when a strong n-p interaction is used. On the other hand, effect of decreasing $\chi_{\nu\pi}$ is also shown in Fig. 3. Calculations with $\alpha = 0.8$ give the rotational- alignments at a lower frequency.

The two-fold effect of a stronger n-p interaction can be clearly seen in the band diagram. When using the factor $\alpha = 1.3$, the rotational behavior of both neutron and proton 2-qp bands are modified so that they cross the g-band at a higher spin and with a smaller crossing angle. In addition, the stronger n-p force acts between the bands resulting in a smoother behavior for the yrast states.

For the other two $N = Z$ nuclei ^{76}Sr and ^{80}Zr in Fig. 3, it can been seen that the effect of the n-p interaction for the band-crossing region seems to have an isotonic dependence. It is observed that increasing the strength $\chi_{\nu\pi}$ by the same amount for ^{76}Sr and ^{80}Zr does not lead to the same pronounced effect as in ^{72}Kr. In fact, varying the factor α from 1.0 to 1.3, we obtain similar smooth curves as our original one [9] for ^{76}Sr, which has already reproduced data well.

It is remarkable that for ^{72}Kr these rather different rotational-alignment features are obtained as a result of different emphasis on the neutron-neutron, proton-proton, and neutron-proton interactions. Therefore empirically increasing the strength of the n-p QQ interaction appears to mimic the experimentally trends and simulates the expected enhanced correlations in the np-channel for these $N = Z$ nuclei.

In principle, the residual n-p interaction which is supposed to be important for the $N = Z$ systems should be of pairing type. It has been known from several mean-field studies (see, for example, [19]) that the n-p pairing is non-zero for $N = Z$ nuclei and vanishes for $N \neq Z$ nuclei. However, the n-p pairing contains pairs of higher angular-momenta [8] apart from $J = 1$ pairs. These higher angular-momentum pairs will have a significant particle-hole contribution and may modify, for example, the QQ interaction used in the PSM Hamiltonian. Therefore, increasing the strength of the QQ in the n-p interaction term, as has been shown in the present work, has a physical origin.

In conclusion, the new experimental data on the $N = Z$ nuclei in the mass-80 region have clearly indicated that the rotational-alignment is substantially delayed for these nuclei. This delay cannot be explained using the standard interaction in the projected shell model approach, which has worked well in the high-spin description for a broad range of nuclei. The quadrupole-pairing interaction, which explains many of the delay in the rotational-alignment for $N \neq Z$ nuclei, does not produce the substantial delay observed in the present $N = Z$ examples. It has been shown that the n-p residual interaction may be responsible for this delay. Our calculations have clearly demonstrated that increasing the strength of the n-p interaction term in the two-body Hamiltonian significantly delays the rotational-alignment. However, to make a quantitative comparison with the experimental data, the renormalization of the interaction for the $N = Z$ nuclei has to be obtained by employing explicitly the n-p pairing term. This work is now under progress and the results will be published in the future.

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FIG. 1. Moments of inertia $\Im^{(1)} = \frac{2I-1}{E(I)-E(I-2)}(\hbar^2/MeV)$ as function of ω^2 with $\omega = \frac{E(I)-E(I-2)}{2}(MeV/\hbar)$. Theoretical results with the standard interaction in the PSM are compared with the experimental data. The experimental data are taken from ref. [1] for ^{72}Kr, ref. [4] for ^{74}Kr, and ref. [11] for ^{76}Kr.

FIG. 2. Moments of inertia $\Im^{(1)} = \frac{2I-1}{E(I)-E(I-2)}(\hbar^2/MeV)$ as function of ω^2 with $\omega = \frac{E(I)-E(I-2)}{2}(MeV/\hbar)$. Theoretical results with various quandrupole pairing strengths are compared with the ^{72}Kr data. The experimental data are taken from ref. [1].

FIG. 3. Moments of inertia $\Im^{(1)} = \frac{2I-1}{E(I)-E(I-2)}(\hbar^2/MeV)$ as function of ω^2 with $\omega = \frac{E(I)-E(I-2)}{2}(MeV/\hbar)$. Theoretical results with various n-p interaction strengths are presented together with the experimental data for three $N = Z$ nuclei, ^{72}Kr, ^{76}Sr and ^{80}Zr. The experimental data are taken from ref. [1].